Worksheet for Sections 6.1 – 6.2
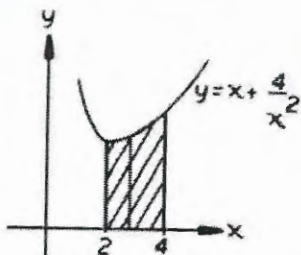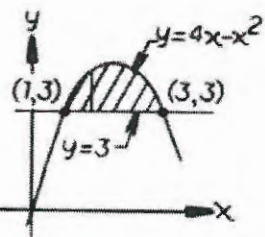
I 1. Find the area of the region enclosed by $y = x + \dfrac{4}{x^2}$, the x-axis, x = 2, and x = 4.

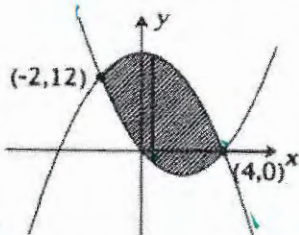$y = x + \dfrac{4}{x^2}$

$\boxed{7}$

2. Find the area of the region enclosed by $y = 4x - x^2$, and y = 3.

$y = 4x - x^2$

(1,3)   (3,3)

$y = 3$

$A = \displaystyle\int_1^3 (4x - x^2 - 3)\,dx = \left[\dfrac{4x^2}{2} - \dfrac{x^3}{3} - 3x\right]_1^3$

$\left((18 - 9 - 9) - \left(2 - \dfrac{1}{3} - 3\right)\right) = \boxed{\dfrac{4}{3}}$

3. Find the area of the region enclosed by $y = x^2 - 4x$ and $y = 16 - x^2$.

(-2,12)   (4,0)

$\displaystyle\int_{-2}^{4} \left(16 - x^2 - (x^2 - 4x)\right)dx = \int_{-2}^{4}(16 - 2x^2 + 4x)\,dx$

$\left[16x - \dfrac{2x^3}{3} + \dfrac{4x^2}{2}\right]_{-2}^{4} = (64 - \dfrac{128}{3} + 32) - (-32 + \dfrac{16}{3} + 8)$
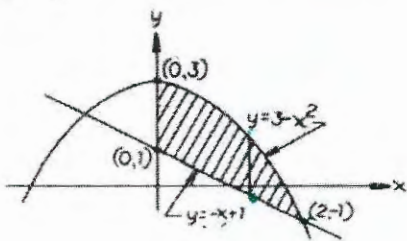
$96 - \dfrac{128}{3} + 24 - \dfrac{16}{3}$

$120 - \dfrac{144}{3} = 120 - 48$

$\boxed{72}$

I 4. Find the area of the region enclosed by $x = y^2 - 4y$ and $x = y$.

$x = y^2 - 4y$ (5,5)

$x = y$

(0,0)
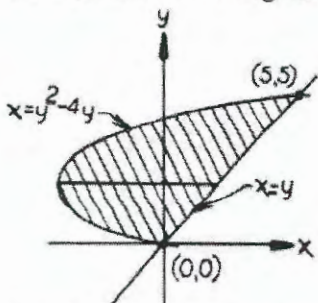
$\boxed{\dfrac{125}{6}}$

$\displaystyle\int_0^5 \left[y - (y^2 - 4y)\right]dy$

$\displaystyle\int_0^5 (y - y^2 + 4y)\,dy = \int_0^5 (5y - y^2)\,dy$

$\left[\dfrac{5y^2}{2} - \dfrac{y^3}{3}\right]_0^5 = \left(\dfrac{125}{2} - \dfrac{125}{3}\right) - (0) = \dfrac{375 - 250}{6}$

$\boxed{\dfrac{125}{6}}$

5. Find the area of the region enclosed by $y = 3 - x^2$ and $y = -x + 1$ between x = 0 and x = 2.

(0,3)

$y = 3 - x^2$

(0,1)

$y = -x + 1$ (2,-1)

$\displaystyle\int_0^2 \left(3 - x^2 - (-x + 1)\right)dx = \int_0^2 (3 - x^2 + x)\,dx$

wait: $\int_0^2 (2 - x^2 + x)\,dx$

$\left[2x - \dfrac{x^3}{3} + \dfrac{x^2}{2}\right]_0^2$

$4 - \dfrac{8}{3} + \dfrac{4}{2} = 6 - \dfrac{8}{3} = \dfrac{18}{3} - \dfrac{8}{3} = \boxed{\dfrac{10}{3}}$

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6. Find the area of the region enclosed by $y = x^2 - 4x$ and $y = 2x - x^2$.
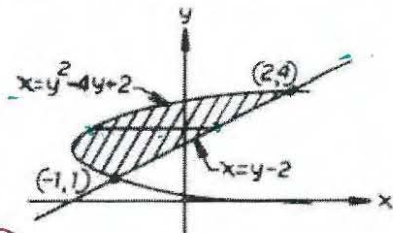
(0,0)

(3,-3)
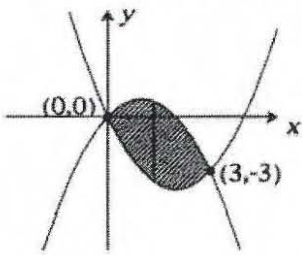
$2x - x^2 - x^2 + 4x$

$\int_0^3 (-2x^2 + 6x)\,dx$

$\left[-\frac{2x^3}{3} + \frac{36x^2}{2} \right]_0^3$

$(-18 + 27) - 0$

$\boxed{9}$

7. Find the area of the region enclosed by $x = y^2 - 4y + 2$ and $x = y - 2$.

$x = y^2 - 4y + 2$ (2,4)

(-1,1) $x = y - 2$

$y - 2 - y^2 + 4y - 2$

$\int_1^4 (-y^2 + 5y - 4)\,dy$
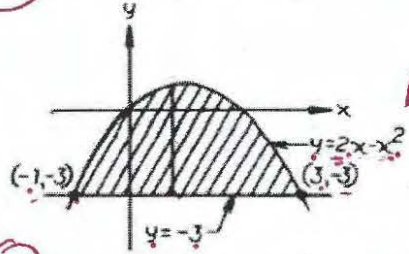
$\left[-\frac{y^3}{3} + \frac{5y^2}{2} - 4y \right]_1^4$

$\left(\frac{-64}{3} + \frac{80}{2} - 16 \right) - \left(-\frac{1}{3} + \frac{5}{2} - 4 \right)$

$24 - \frac{64}{3} + \frac{1}{3} - \frac{5}{2} + 4$

$28 - 21 - \frac{5}{2} = 7 - \frac{5}{2}$ $\boxed{\frac{9}{2}}$

8. Find the area of the region enclosed by $y = 2x - x^2$ and $y = -3$.

(-1,-3)   (3,-3)

$y = 2x - x^2$

$y = -3$

$A = \int_{-1}^3 (2x - x^2 + 3)\,dx$

$\left[\frac{2x^2}{2} - \frac{x^3}{3} + 3x \right]_{-1}^3$

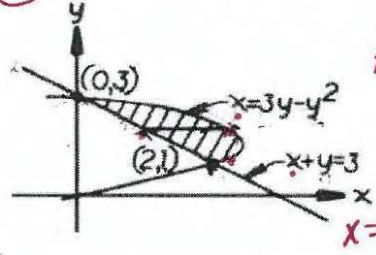$(9 - 9 + 9) - \left(1 + \frac{1}{3} + 3 \right)$

$9 - 1 - \frac{1}{3} + 3$

$11 - \frac{1}{3} = 10\,\frac{2}{3}$

$\boxed{\frac{32}{3} \text{ units}^2}$

9. Find the area of the region enclosed by $x = 3y - y^2$ and $x + y = 3$.

(0,3) $x = 3y - y^2$

(2,1) $x + y = 3$

$x = 3 - y$

$A = \int_1^3 \left[(3y - y^2) - (3 - y) \right]\,dy$

$3y - y^2 - 3 + y$

$\int_1^3 (4y - y^2 - 3)\,dy$

10. Use the disk method to find the volume of the solid that results when the area of the region enclosed by $y = x^2$, $x = 0$, and $y = 4$ is revolved about the y-axis.

y=4

(2,4)

$y = x^2$

$A = \left[\frac{4y^2}{2} - \frac{y^3}{3} - 3y \right]_1^3$

$(18 - 9 - 9) - \left(2 - \frac{1}{3} - 3 \right)$ $\boxed{9}$

$-2 + \frac{1}{3} + 3 = 1 + \frac{1}{3} = \boxed{\frac{4}{3} \text{ units}^2}$

10     $\boxed{8\pi}$